APPENDIX.

A.

The following memoirs by the author, bearing on Heredity, have been variously utilised in this volume:

Experiments in Pangenesis. *Proc. Royal Soc.*, No. 127, 1871, p. 393.

Blood Relationship. *Proc. Royal Soc.*, No. 136, 1872, p. 394.

A Theory of Heredity. *Journ. Anthropol. Inst.*, 1875, p. 329.

Statistics by Intercomparison. *Phil. Mag.*, Jan. 1875.

*On the Probability of the Extinction of Families. *Journ. Anthropol. Inst.*, 1875.

Typical laws of Heredity. *Journ. Royal Inst.*, Feb. 1877.

*Geometric Mean in Vital and Social Statistics. *Proc. Royal Soc.*, No. 198, 1879. See subsequent memoir by Dr. Macalister.

Address to Anthrop. Section British Association at Aberdeen. *Journ. Brit. Assoc.*, 1885.

Regression towards Mediocrity in Hereditary Stature. *Journ. Anthropol. Inst.*, 1885.

Presidential Addresses to Anthropol. Inst., 1885, 6 and 7.

Family Likeness in Stature. *Proc. Royal Soc.*, No. 242, 1886.

Family Likeness in Eye-colour. *Proc. Royal Soc.*, No. 245, 1886.

*Good and Bad Temper in English Families. *Fortnightly Review*, July, 1887.

Pedigree Moth Breeding. *Trans. Entomolog. Soc.*, 1887. See also subsequent memoir by Mr. Merrifield, and another read by him, Dec. 1887.

Those marked with an asterisk (*) are reprinted with slight revision in the Appendices F, D, and E.

WORKS ON HEREDITY BY THE AUTHOR.

(Published by Messrs. Macmillan & Co.)

Hereditary Genius. 1869.
English Men of Science. 1874.
Inquiries into Human Faculty. 1883.

Record of Family Faculties.[1] 1884. 2*s.* 6*d.*
Life History Album[2] (*edited* by F. Galton). 1884. 3*s.* 6*d.* and 4*s.* 6*d.*

[1] The Record of Family Faculties consists of Tabular Forms and Directions for entering Data, with an Explanatory Preface. It is a large thin quarto book of seventy pages, bound in limp cloth. The first part of it contains a preface, with explanation of the object of the work and of the way in which it is to be used. The rest consists of blank forms, with printed questions and blank spaces to be filled with writing. The Record is designed to facilitate the orderly collection of such data as are important to a family from an hereditary point of view. It allots equal space to every direct ancestor in the nearer degrees, and is supposed to be filled up in most cases by a parent, say the father of a growing family. If he takes pains to make inquiries of elderly relatives and friends, and to seek in registers, he will be able to ascertain most of the required particulars concerning not only his own parents, but also concerning his four grandparents; and he can ascertain like particulars concerning those of his wife. Therefore his children will be provided with a large store of information about their two parents, four grandparents, and eight great-grandparents, which form the whole of their fourteen nearest ancestors. A separate schedule is allotted to each of them. Space is afterwards provided for the more important data concerning many at least, of the brothers and sisters of each direct ancestor. The schedules are followed by Summary Tables, in which the distribution of any characteristic throughout the family at large may be compendiously exhibited.

[2] The Life History Album was prepared by a Sub-Committee of the Collective Investigation Committee of the British Medical Association. It is designed to serve as a continuous register of the principal biological facts in the life of its owner. The book begins with a few pages of explanatory remarks, followed by tables and charts. The first table is to contain a brief medical history of each member of the near ancestry of the owner. This is followed by printed forms on which the main facts of the owner's growth and development from birth onwards may be registered, and by charts on which measurements may be laid down at appropriate intervals and compared with the curves of normal growth. Most of the required data are such as any intelligent person is capable of recording; those that refer to illnesses should be brief and technical, and ought to be filled up by the medical attendant. Explanations are given of the most convenient tests of muscular force, of keenness of eyesight and hearing, and of the colour sense. *The 4s. 6d. edition contains a card of variously coloured wools to* test the sense of colour.

*** These two works pursue similar objects of personal and scientific utility, along different paths. The Album is designed to lay the foundation of a practice of maintaining trustworthy life-histories that shall be of medical service in after-life to the person who keeps them. The Record shows how the life histories of members of the same family may be collated and used to forecast the development in mind and body of the younger generation of that family. Both works are intended to promote the registration of a large amount of information that has hitherto been allowed to run to waste in oblivion, instead of accumulating and forming stores of recorded experience for future personal use, and from which future inquirers into heredity may hope to draw copious supplies.

B.

PROBLEMS BY J. D. HAMILTON DICKSON, FELLOW AND TUTOR OF ST. PETER'S COLLEGE, CAMBRIDGE.

(*Reprinted from Proc. Royal Soc., No.* 242, 1886, *p.* 63.)

Problem 1.—A point P is capable of moving along a straight line P′OP, making an angle $\tan^{-1}\frac{2}{3}$ with the axis of y, which is drawn through O the mean position of P; the probable error of the projection of P on Oy is 1·22 inch: another point p, whose mean position at any time is P, is capable of moving from P parallel to the axis of x (rectangular co-ordinates) with a probable error of 1·50 inch. To discuss the "surface of frequency" of p.

1. Expressing the "surface of frequency" by an equation in x, y, z, the exponent, with its sign changed, of the exponential which appears in the value of z in the equation of the surface is, save as to a factor,

$$\frac{y^2}{(1\cdot22)^2}+\frac{(3x-2y)^2}{9(1\cdot50)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

hence all sections of the "surface of frequency" by planes parallel to the plane of xy are ellipses, whose equations may be written in the form,

$$\frac{y}{(1\cdot22)^2}+\frac{(3x-2y)^2}{9(1\cdot50)^2}=\text{C, a constant} \quad . \quad . \quad . \quad (2)$$

2. Tangents to these ellipses parallel to the axis of y are found,

by differentiating (2) and putting the coefficient of dy equal to zero, to meet the ellipses on the line,

$$\left.\begin{aligned} &\frac{y}{(1\cdot22)^2} - 2\frac{3x-2y}{9(1\cdot50)^2} = 0, \\ \text{that is} \quad &\frac{y}{x} = \frac{\frac{6}{9(1\cdot50)^2}}{\frac{1}{(1\cdot22)^2}+\frac{4}{9(1\cdot50)^2}} = \frac{6}{17\cdot6} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \quad (3)$$

or, approximately, on the line $y = \frac{1}{3}x$. Let this be the line OM. (See Fig. 11, p. 101.)

From the nature of conjugate diameters, and because P is the mean position of p, it is evident that tangents to these ellipses parallel to the axis of x meet them on the line $x = \frac{2}{3}y$, viz., on OP.

3. Sections of the "surface of frequency" parallel to the plane of xz, are, from the nature of the question, evidently curves of frequency with a probable error 1·50, and the locus of their vertices lies in the plane z OP.

Sections of the same surface parallel to the plane of yz are got from the exponential factor (1) by making x constant. The result is simplified by taking the origin on the line OM. Thus putting $x = x_1$ and $y = y_1 + y'$, where by (3)

$$\frac{y_1}{(1\cdot22)^2} = 2\frac{3x_1 - 2y_1}{9(1\cdot50)^2} = 0$$

the exponential takes the form

$$\left\{\frac{1}{(1\cdot22)^2}+\frac{4}{9(1\cdot50)^2}\right\}y'^2 + \left\{\frac{y_1^2}{(1\cdot22)^2}+\frac{(3x_1-2y_1)^2}{9(1\cdot50)^2}\right\} \quad . \; . \; . \quad (4)$$

whence, if e be the probable error of this section,

$$\left.\begin{aligned} &\frac{1}{e^2} = \frac{1}{(1\cdot22)^2} + \frac{4}{9(1\cdot50)^2} \\ \text{or [on referring to (3)]} \quad &e = 1\cdot50\sqrt{\frac{9}{17\cdot6}} \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \quad (5)$$

that is, the probable error of sections parallel to the plane of yz is nearly $\frac{1}{\sqrt{2}}$ times that of those parallel to the plane of xz, and the locus of their vertices lies in the plane zOM.

It is important to notice that all sections parallel to the same co-ordinate plane have the same probable error.

4. The ellipses (2) when referred to their principal axes become, after some arithmetical simplification,

$$\frac{x'^2}{20 \cdot 68} + \frac{y'^2}{5 \cdot 92} = \text{constant}, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

the major axis being inclined to the axis of x at an angle whose tangent is 0·5014. [In the approximate case the ellipses are $\frac{x'^2}{7} + \frac{y^2}{2} = \text{const.}$, and the major axis is inclined to the axis of x at an angle $\tan^{-1}\frac{1}{2}$.]

5. The question may be solved in general terms by putting YON $= \theta$, XOM $= \phi$, and replacing the probable errors 1·22 and 1·50 by a and b respectively; then the ellipses (2) are,

$$\frac{y^2}{a^2} + \frac{(x - y\tan\theta)^2}{b^2} = C. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

equation (3) becomes

$$\left.\begin{aligned} &\frac{y^2}{a^2} + \tan\theta\frac{x - y\tan\theta}{b^2} = 0 \\ \text{or} \quad &\frac{y}{x} = \tan\phi = \frac{a^2\tan\theta}{b^2 + a^2\tan^2\theta} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and (5) becomes

$$\frac{1}{e^2} = \frac{1}{a^2} + \frac{\tan^2\theta}{b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

whence

$$\frac{\tan\phi}{\tan\theta} = \frac{e^2}{b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

If c be the probable error of the projection of p's whole motion on the plane of xz, then

$$c^2 = a^2\tan^2\theta + b^2,$$

which is independent of the distance of p's line of motion from the axis of x. Hence also

$$\frac{\tan\phi}{\tan\theta} = \frac{a^2}{b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Problem 2.—An index q moves under some restraint up and down a bar AQB, its mean position for any given position of the bar being Q ; the bar, always carrying the index with it, moves under some restraint up and down a fixed frame YMY′, the mean position of Q being M : the movements of the index relatively to the bar and of the bar relatively to the frame being quite independent. For any given observed position of q, required the most probable position of Q (which cannot be observed); it being known that the probable error of q relatively to Q in all positions is b, and that of Q relatively to M is c. The ordinary law of error is to be assumed.

If in any one observation, $MQ = x$, $Qq = y$, then the law of error requires

$$\frac{x^2}{c^2} + \frac{y^2}{b^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

to be a minimum, subject to the condition

$$x + y = a, \text{ a constant.}$$

Hence we have at once, to determine the most probable values of x', y',

$$\frac{x'}{c^2} = \frac{y'}{b^2} = \frac{a}{b^2 + c^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and the most probable position of Q, measured from M, when q's observed distance from M is a, is

$$\frac{c^2}{b^2 + c^2}\, a.$$

It also follows at once that the probable error v of Q (which may be obtained by substituting $a - x$ for y in (12)) is given by

$$\frac{1}{v^2} = \frac{1}{c^2} + \frac{1}{b^2}, \text{ or } v = \frac{bc}{\sqrt{b^2 + c^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

which it is important to notice, is the same for all values of a.

C.

EXPERIMENTS ON SWEET PEAS BEARING ON THE LAW OF REGRESSION.

The reason why Sweet Peas were chosen, and the methods of selecting and planting them are described in Chapter VI., p. 79. The following Table justifies their selection by the convenient and accurate method of weighing, as equivalent to that of measuring them. It will be seen that within the limits of observed variation a difference of 0·172 grain in weight corresponds closely to an average difference of 0·01 inch in diameter.

Table 1.

Comparison of Weights of Sweet Peas with their Diameters.

Distinguishing letter of seed.	Weight of one seed in grains. — Common difference = 0·172 grain.	Length of row of 100 seeds in inches.	Diameter of one seed in hundredths of inch. — Common difference = 0·01 inch.
K	1·750	21·0	21
L	1·578	20·2	20
M	1·406	19·2	19
N	1·234	17·9	18
O	1·062	17·0	17
P	·890	16·1	16
Q	·718	15·2	15

The results of the experiment are given in Table 2; its first and last columns are those that especially interest us; the remaining columns showing how these two were obtained.

It will be seen that for each increase of one unit on the part of the parent seed, there is a mean increase of only one-third of a unit in the filial seed; and again that the mean filial seed resembles the parental when the latter is about 15·5 hundredths of an inch in diameter. Taking 15·5 as the point towards which Filial Regression points, whatever may be the parental deviation from that point, the mean Filial Deviation will be in the same direction, but only one-third as much.

TABLE 2.

PARENT SEEDS AND THEIR PRODUCE.

The proportionate number of sweet peas of different sizes, produced by parent seeds also of different sizes, are given below. The measurements are those of their mean diameters, in hundredths of an inch.

Diameter of Parent Seed.	Diameters of Filial Seeds.								Total.	Mean Diameter of Filial Seeds.	
	Under 15.	15–	16–	17–	18–	19–	20–	Above 21–		Observed	Smoothed
21	22	8	10	18	21	13	6	2	100	17·5	17·3
20	23	10	12	17	20	13	3	2	100	17·3	17·0
19	35	16	12	13	11	10	2	1	100	16·0	16 6
18	34	12	13	17	16	6	2	0	100	16·3	16·3
17	37	16	13	16	13	4	1	0	100	15·6	16·0
16	34	15	18	16	13	3	1	0	100	16·0	15·7
15	46	14	9	11	14	4	2	0	100	15·3	15·4

This point is so low in the scale, that I possess less evidence than I desired to prove the bettering of the produce of very small seeds. The seeds smaller than Q were such a miserable set that I could hardly deal with them. Moreover, they were very infertile. It did, however, happen that in a few of the sets some of the Q seeds turned out very well.

If I desired to lay much stress on these experiments, I could make my case considerably stronger by going minutely into other details, including confirmatory measurements of the foliage and length of pod, but I do not care to do so.

D.

GOOD AND BAD TEMPER IN ENGLISH FAMILIES.[1]

ONE of the questions put to the compilers of the Family Records spoken of in page 72, referred to the "Character and Temperament" of the persons described. These were distributed through

[1] Reprinted after slight revision from *Fortnightly Review*, July, 1887.

three and sometimes four generations, and consisted of those who lay in the main line of descent, together with their brothers and sisters.

Among the replies, I find that much information has been incidentally included concerning what is familiarly called the "temper" of no less than 1,981 persons. As this is an adequate number to allow for many inductions, and as temper is a strongly marked characteristic in all animals; and again, as it is of social interest from the large part it plays in influencing domestic happiness for good or ill, it seemed a proper subject for investigation.

The best explanation of what I myself mean by the word "temper" will be inferred from a list of the various epithets used by the compilers of the Records, which I have interpreted as expressing one or other of its qualities or degrees. The epithets are as follows, arranged alphabetically in the two main divisions of good and bad temper:—

Good temper.—Amiable, buoyant, calm, cool, equable, forbearing, gentle, good, mild, placid, self-controlled, submissive, sunny, timid, yielding. (15 epithets in all.)

Bad temper.—Acrimonious, aggressive, arbitrary, bickering, capricious, captious, choleric, contentious, crotchety, decisive, despotic, domineering, easily offended, fiery, fits of anger, gloomy, grumpy, harsh, hasty, headstrong, huffy, impatient, imperative, impetuous, insane temper, irritable, morose, nagging, obstinate, odd-tempered, passionate, peevish, peppery, proud, pugnacious, quarrelsome, quick-tempered, scolding, short, sharp, sulky, sullen, surly, uncertain, vicious, vindictive. (46 epithets in all.)

I also grouped the epithets as well as I could, into the following five classes: 1, mild; 2, docile; 3, fretful; 4, violent; 5, masterful.

Though the number of epithets denoting the various kinds of bad temper is three times as large as that used for the good, yet the number of persons described under the one general head is about the same as that described under the other. The first set of data that I tried, gave the proportion of the good to the bad-tempered as 48 to 52; the second set as 47 to 53. There is little difference between the two sexes in the frequency of good and bad temper, but that little is in favour of the women, since about 45 men are recorded as good-tempered for every 55 who are bad, and conversely 55 women as good-tempered for 45 who are bad.

I will not dwell on the immense amount of unhappiness, ranging from family discomfort down to absolute misery, or on the breaches of friendship that must have been occasioned by the cross-grained, sour, and savage dispositions of those who are justly labelled by some of the severer epithets; or on the comfort, peace, and good-will diffused through domestic circles by those who are rightly described by many of the epithets in the first group. We can hardly, too, help speculating uneasily upon the terms that our own relatives would select as most appropriate to our particular selves. But these considerations, interesting as they are in themselves, lie altogether outside the special purpose of this inquiry.

In order to ascertain the facts of which the above statistics are a brief summary, I began by selecting the larger families out of my lists, namely, those that consisted of not less than four brothers or sisters, and by noting the persons they included who were described as good or bad-tempered; also the remainder about whose temper nothing was said either one way or the other, and whom perforce I must call neutral. I am at the same time well aware that, in some few cases a tacit refusal to describe the temper should be interpreted as reticence in respect to what it was thought undesirable even to touch upon.

I found that out of a total of 1,361 children, 321 were described as good-tempered, 705 were not described at all, and 342 were described as bad-tempered. These numbers are nearly in the proportion of 1, 2, and 1, that is to say, the good are equal in number to the bad-tempered, and the neutral are just as numerous as the good and bad-tempered combined.

The equality in the total records of good and bad tempers is an emphatic testimony to the correct judgments of the compilers in the choice of their epithets, for whenever a group has to be divided into three classes, of which the second is called neutral, or medium, or any other equivalent term, its nomenclature demands that it should occupy a strictly middlemost position, an equal number of contrasted cases flanking it on either hand. If more cases were recorded of good temper than of bad, the compilers would have laid down the boundaries of the neutral zone unsymmetrically, too far from the good end of the scale of temper, and too near the bad end. If the number of cases of bad temper exceeded that of the good, the error would have been in the opposite direction. But it appears, on the whole, that the compilers of the records have erred neither to the right hand nor to the left. So far, therefore, their judgments are shown to be correct.

Next as regards the proportion between the number of those who rank as neutrals to that of the good or of the bad. It was recorded as 2 to 1; is that the proper poportion? Whenever the nomenclature is obliged to be somewhat arbitrary, a doubtful term should be interpreted in the sense that may have the widest suitability. Now a large class of cases exist in which the interpretation of the word neutral is fixed. It is that in which the three grades of magnitude are conceived to result from the various possible combinations of two elements, one of which is positive and the other negative, such as good and bad, and which are supposed to occur on each occasion at haphazard, but in the long run with equal frequency. The number of possible combinations of the two elements is only four, and each of these must also in the long run occur with equal frequency. They are: 1, both positive; 2, the first positive, the second negative; 3, the first negative, the second positive; 4, both negative. In the second and third of these combinations the negative counterbalances the positive, and the result is neutral. Therefore the proportions in which the several events of good, neutral, and bad would occur is as 1, 2, and 1. These proportions further commend themselves on the ground that the whole body of cases is thereby divided into two main groups, equal in number, one of which includes all neutral or medium cases, and the other all that are exceptional. Probably it was this latter view that was taken, it may be half unconsciously, by the compilers of the Records. Anyhow, their entries conform excellently to the proportions specified, and I give them credit for their practical appreciation of what seems theoretically to be the fittest standard. I speak, of course, of the Records taken as a whole; in small groups of cases the proportion of the neutral to the rest is not so regular.

The results shown in Table I. are obtained from all my returns. It is instructive in many ways, and not least in showing to a statistical eye how much and how little value may reasonably be attached to my materials. It was primarily intended to discover whether any strong bias existed among the compilers to spare the characters of their nearest relatives. In not a few cases they have written to me, saying that their records had been drawn up with perfect frankness, and earnestly reminding me of the importance of not allowing their remarks to come to the knowledge of the persons described. It is almost needless to repeat what I have published more than once already, that I treat the Records quite confidentially. I have left written instructions that in case of my death they should all be destroyed unread, except where I have left a note to say that the compiler wished them returned. In some instances I know that the Records were compiled by a sort of family council, one of its members acting as secretary; but I doubt much whether it often happened that the Records were known to many of the members of the family in their complete form. Bearing these and other considerations in mind, I thought the best test for bias would be to divide the entries into two contrasted groups, one including those who figured in the pedigrees as either father, mother, son, or daughter—that is to say, the compiler and those who were very nearly related to him—and the other including the uncles and aunts on both sides.

TABLE 1.

DISTRIBUTION OF TEMPER IN FAMILIES (per cents.)

Relationships.	1. Mild.	2. Docile.	3. Fretful.	4. Violent.	5. Masterful.	Total.	No. of cases observed.
a. Fathers and Sons	35	12	32	12	9	100	188
b. Mothers and Daughters	39	18	31	8	4	100	179
c. Uncles....................	32	13	25	18	12	100	272
d. Aunts.....................	39	14	29	9	9	100	238
a + *b*. Direct line..........	74	30	63	20	13	200	367
c + *d*. Collaterals..........	71	27	54	27	21	200	510
	Good.		Bad Temper.				
a + *b*. Direct line..........	104		96			200	367
c + *d*. Collaterals..........	98		102			200	510

On comparing the entries, especially the summaries in the lower lines of the Table, it does not seem that the characters of near relatives are treated much more tenderly than those of the more remote. There is little indication of the compilers having been biased by affection, respect, or fear. More cases of a record being left blank when a bad temper ought to have been recorded, would probably occur in the direct line, but I do not see how this could be tested. An omission may be due to pure ignorance; indeed I find it not uncommon for compilers to know very little of some of their uncles or aunts. The Records seem to be serious and careful compositions, hardly ever used as vehicles for personal animosity, but written in much the same fair frame of mind that most people force themselves into when they write their wills.

TABLE 2.

COMBINATIONS OF TEMPER IN MARRIAGE (per cents.).

Tempers of Husbands.	A.—Observed Pairs.					B.—Haphazard Pairs.				
	Tempers of Wives.					Tempers of Wives.				
	Good.		Bad Tempers.			Good.		Bad Tempers.		
	1	2	3	4	5	1	2	3	4	5
Good 1	6	10	9	6	2	13	5	10	3	2
,, ... 2	4	2	5	2	—	5	2	4	1	1
Bad 3	14	4	9	3	2	11	5	8	2	2
,, ... 4	7	3	3	2	1	6	2	5	1	1
,, ... 5	3	—	2	—	1	4	2	3	1	1
Good...........	22		24			25		21		
Bad............	31		23			30		24		

The sexes are separated in the Table, to show the distribution of the five classes of temper among them severally. There is a large proportion of the violent and masterful among the men, of the fretful, the mild, and the docile among the women. On adding the entries it will be found that the proportion of those who fall within the several classes are 36 per cent. of mild-tempered, 15 per cent. of docile, 29 per cent. of fretful, 12 per cent of violent, 8 per cent. of masterful.

The importance assigned in marriage-selection to good and bad temper is an interesting question, not only from its bearing on domestic happiness, but also from the influence it may have in promoting or retarding the natural good temper of our race, assuming, as we may do for the moment, that temper is hereditary. I cannot deal with the question directly, but will give some curious facts in Table II. that throw indirect light upon it. There a comparison is made of (A) the actual frequency of marriage between persons, each of the various classes of temper, with (B) the calculated frequency according to the laws of chance, on the supposition that there had been no marriage-selection at all, but that the pairings, so far as temper is concerned, had been purely at haphazard. There are only 111 marriages in my lists in which the tempers of both parents are recorded. On the other hand, the number of possible combinations in couples of persons who belong to the five classes of temper is very large, so I make the two groups comparable by reducing both to percentages.

It will be seen that with two apparent exceptions in the upper left-hand corners of either Table (of 6 against 13, and of 10 against 5), there are no indications of predilection for, or avoidance of marriage between persons of any of the five classes, *but that the* figures taken from observation run as closely with those derived through calculation, as could be expected from the small number of observations. The apparent exceptions are that the percentage of mild-tempered men who marry mild-tempered women is only 6, as *against 13 calculated by the laws of chance, and that those who* marry docile wives are 10, as against a calculated 5. There is little difference between mildness and docility, so we may throw these entries together without much error, and then we have 6 and 10, or 16, as against 13 and 5, or 18, which is a close approximation. We may compare the frequency of marriages between persons of like temper in each of the five classes by reading the Table diagonally. They are as (6), 2, 9, 2, 1, in the observed cases, against (13), 2, 8, 1, 1, in the calculated ones; here the irregularity of the 6 and 13, which are put in brackets for distinction sake, is conspicuous. Elsewhere there is not the slightest indication of a dislike in persons of similar tempers, whether mild, docile, fretful, violent, or masterful, to marry one another. The large initial figures 6 and 13 catch the eye, and at a first glance impress themselves unduly on the imagination, and might lead to erroneous speculations about mild tempered persons, perhaps that they find one another rather insipid; but the reasons I have given, show conclusively that the recorded rarity of the marriages between mild-tempered persons is only apparent. Lastly, if we disregard the five smaller classes and attend only to the main divisions of good and bad temper, there does not appear to be much bias for, or against, the marriage of good or bad-tempered persons in their own or into the opposite division.

The admixture of different tempers among the brothers and sisters of the same family is a notable fact, due to various causes which act in different directions. It is best to consider them before we proceed to collect evidence and attempt its interpretation. It becomes clear enough, and may be now taken for granted, that the tempers of progenitors do not readily blend in the offspring, but that some of the children take mainly after one of them, some after another, but with a few threads, as it were, of various ancestral tempers woven in, which occasionally manifest themselves. If no other influences intervened, the tempers of the children in the same family would on this account be almost as varied as those of their ancestors; and these, as we have just seen, married at haphazard, so far as their tempers were concerned; therefore the numbers of good and bad children in families would be regulated by the same laws of chance that apply to a gambling table. But there are other influences to be considered. There is a well-known tendency to family likeness among brothers and sisters, which is due, not to the blending of ancestral peculiarities, but to the prepotence of one of the progenitors, who stamped more than his or her fair share of qualities upon the descendants. It may be due also to a familiar occurrence that deserves but has not yet received a distinctive name, namely, where all the children are alike and yet their common likeness cannot be traced to their progenitors. A new variety has come into existence through a process that affects the whole Fraternity and may result in an unusually stable variety (see Chapter III.). The most strongly marked family type that I have personally met with, first arose simultaneously in the three brothers of a family who transmitted their peculiarities with unusual tenacity to numerous descendants through at least four generations. Other influences act in antagonism to the foregoing; they are the events of domestic life, which instead of assimilating tempers tend to accentuate slight differences in them. Thus if some members of a family are a little submissive by nature, others who are naturally domineering are tempted to become more so. Then the acquired habit of dictation in these reacts upon the others and makes them still more submissive. In the collection I made of the histories of twins who were closely alike, it was most commonly said that one of the twins was guided by the other. I suppose that after their many childish struggles for supremacy, each finally discovered his own relative strength of character, and thenceforth the stronger developed into the leader, while the weaker contentedly subsided into the position of being led. Again, it is sometimes observed that one member of an otherwise easygoing family, discovers that he or she may exercise considerable power by adopting the habit of being persistently disagreeable whenever he or she does not get the first and best of everything. Some wives contrive to tyrannise over husbands who are mild and sensitive, who hate family scenes and dread the disgrace attending them, by holding themselves in readiness to fly into a passion whenever their wishes are withstood. They thus acquire a habit of "breaking out," to use a term familiar to the warders of female prisons and lunatic asylums; and though their relatives and connections would describe their tempers by severe epithets, yet if they had married masterful husbands their characters might have developed more favourably.

To recapitulate briefly, one set of influences tends to mix good and bad tempers in a family at haphazard; another set tends to assimilate them, so that they shall all be good or all be bad; a third set tends to divide each family into contrasted portions. We have now to ascertain the facts and learn the results of these opposing influences.

In dealing with the distribution of temper in Fraternities,[1] we

[1] A Fraternity consists of the brothers of a family, and of the sisters after the qualities of the latter have been transmuted to their Male Equivalents; but as no change in the Female values seems really needed, so none has been made in respect to Temper.

can only make use of those in which at least two cases of temper are recorded; they are 146 in number. I have removed all the cases of neutral temper, treating them as if they were non-existent, and dealing only with the remainder that are good or bad. We have next to eliminate the haphazard element. Beginning with Fraternities of two persons only, either of whom is just as likely to be good as bad tempered, there are, as we have already seen, four possible combinations, resulting in the proportions of 1 case of both good, 2 cases one good and one bad, and one case of both bad. I have 42 such Fraternities, and the observed facts are that in 10 of them both are good tempered, in 20 one is good and one bad, and in 12 both are bad tempered. Here only a trifling and untrustworthy difference is found between the observed and the haphazard distribution, the other conditions having neutralised each other. But when we proceed to larger Fraternities the test becomes shrewder, and the trifling difference already observed becomes more marked, and is at length unmistakable. Thus the successive lines of Table III. show a continually increasing divergence between the observed and the haphazard distribution of temper, as the Fraternities increase in size. A compendious com-

TABLE 3.

DISTRIBUTION OF TEMPER IN FRATERNITIES.

		A.—Observed.			B.—Haphazard.		
Number in each Fraternity.	Number of Fraternities.	All *good*-tempers.	*Intermediate* cases.	All *bad*-tempers.	All *good*-tempers.	*Intermediate* cases.	All *bad*-tempers.
2	42	10	20	12	10	21	11
3	55	11	15 21	8	7	20 21	7
4	29	5	6 9 8	1	2	8 12 8	2
5	6	1	0 2 1 0	2	0	1 2 2 1	0
6	14	1	0 1 3 3 2	4	0	2 4 5 4 2	0
4 to 6	49	7		7	2		2

parison is made in the bottom line of the Table by adding together the instances in which the Fraternities are from 4 to 6 in number, and in taking only those in which all the members of the Fraternity were alike in temper, whether good or bad. There are 7 + 7, or 14, observed cases of this against 2 + 2, or 4, haphazard cases, found in a total of 49 Fraternities. Hence it follows that the domestic influences that tend to differentiate temper wholly fail to overcome the influences, hereditary and other, that tend to make it uniform in the same Fraternity.

As regards direct evidences of heredity of temper, we must frame our inquiries under a just sense of the sort of materials we have to depend upon. They are but coarse portraits scored with white or black, and sorted into two heaps, irrespective of the gradations of tint in the originals. The processes I have used in discussing the heredity of stature, eye-colour, and artistic faculty, cannot be employed in dealing with the heredity of temper. I must now renounce those refined operations and set to work with ruder tools on my rough material.

The first inquiry will be, Do good-tempered parents have, on the whole, good-tempered children, and do bad-tempered parents have bad-tempered ones? I have 43 cases where both parents are recorded as good-tempered, and 25 where they were both bad-tempered. Out of the children of the former, 30 per cent. were good-tempered and 10 per cent. bad; out of the latter, 4 per cent. were good and 52 per cent. bad-tempered. This is emphatic testimony to the heredity of temper. I have worked out the other less contrasted combinations of parental temper, but the results are hardly worth giving. There is also much variability in the proportions of the neutral cases.

I then attempted, with still more success, to answer the converse question, Do good-tempered Fraternities have, on the whole, good-tempered ancestors, and bad tempered Fraternities bad-tempered ones? After some consideration of the materials, I defined—rightly or wrongly—a good-tempered Fraternity as one in which at least two members were good-tempered and none were bad, and a bad-tempered Fraternity as one in which at least two members were bad-tempered, whether or no any cases of good temper were said to be associated with them. Then, as regards the ancestors, I thought by far the most trustworthy group was that which consisted of the two parents and of the uncles and aunts on both sides. I have thus 46 good-tempered Fraternities with an aggregate of 333 parents, uncles, and aunts; and 71 bad-tempered, with 633 parents, uncles, and aunts. In the former group, 26 per cent. were good tempered and 18 bad; in the latter group, 18 were good-tempered and 29 were bad, the remainder being neutral. These results are almost the exact counterparts of one another, so I seem to have made good hits in framing the definitions. More briefly, we mav say that when the Fraternity is good-tempered as above defined, the number of good-tempered parents, uncles, and aunts, exceeds that of the bad-tempered in the proportion of 3 to 2; and that when the Fraternity is bad-tempered, the proportions are exactly reversed.

I have attempted in other ways to work out the statistics of hereditary tempers, but none proved to be of sufficient value for publication. I can trace no prepotency of one sex over the other in transmitting their tempers to their children. I find clear indications of strains of bad temper clinging to families for three generations, but I cannot succeed in putting them into a numerical form.

It must not be thought that I have wished to deal with temper as if it were an unchangeable characteristic, or to assign more trustworthiness to my material than it deserves. Both these views have been discussed; they are again alluded to to show that they are not dismissed from my mind, and partly to give the opportunity of adding a very few further remarks.

Persons highly respected for social and public qualities may be well-known to their relatives as having sharp tempers under strong but insecure control, so that they "flare up" now and then. I have heard the remark that those who are over-suave in ordinary demeanour have often vile tempers. If this be the case—and I have some evidence of its truth—I suppose they are painfully conscious of their infirmity, and through habitual endeavours to subdue it, have insensibly acquired an exaggerated suavity at the times when their temper is unprovoked. Illness, too, has much influence in affecting the temper. Thus I sometimes come across entries to the effect of, "not naturally ill-tempered, but peevish through illness." Overwork and worry will make even mild-tempered men exceedingly touchy and cross.

The accurate discernment and designation of character is almost beyond the reach of any one, but, on the other hand, a rough estimate and a fair description of its prominent features is easily obtainable; and it seems to me that the testimony of a member of a family who has seen and observed a person in his unguarded moments and under very varied circumstances for many years, is a verdict deserving of much confidence. I shall have fulfilled my object in writing this paper if it leaves a clear impression of the great range and variety of temper among persons of both sexes in the upper and middle classes of English society; of its disregard in Marriage Selection; of the great admixture of its good and bad varieties in the same family; and of its being, nevertheless, as hereditary as any other quality. Also, that although it exerts an immense influence for good or ill on domestic happiness, it seems that good temper has not been especially looked for, nor ill temper especially shunned, as it ought to be in marriage-selection.

E.

THE GEOMETRIC MEAN, IN VITAL AND SOCIAL STATISTICS.[1]

My purpose is to show that an assumption which lies at the basis of the well-known law of "Frequency of Error" is incorrect when applied to many groups of vital and social phenomena, although that law has been applied to them by statisticians with partial success. Next, I will point out the correct hypothesis upon which a Law of Error suitable to these cases ought to be calculated; and subsequently I will communicate a memoir by Mr. (now Dr.) Donald Macalister, who, at my suggestion, has mathematically investigated the subject.

The assumption to which I refer is, that errors in excess or in deficiency of the truth are equally probable; or conversely, that if two fallible measurements have been made of the same object, their

[1] Reprinted, with slight revision, from the *Proceedings of the Royal Society*, No. 198, 1879.

arithmetical mean is more likely to be the true measurement than any other quantity that can be named.

This assumption cannot be justified in vital phenomena. For example, suppose we endeavour to match a tint; Weber's law, in its approximative and simplest form, of Sensation varying as the logarithm of the Stimulus, tells us that a series of tints, in which the quantities of white scattered on a black ground are as 1, 2, 4, 8, 16, 32, &c., will appear to the eye to be separated by equal intervals of tint. Therefore, in matching a grey that contains 8 portions of white, we are just as likely to err by selecting one that has 16 portions as one that has 4 portions. In the first case there would be an error in excess, of 8 units of absolute tint; in the second there would be an error in deficiency, of 4. Therefore, an error of the same magnitude in excess or in deficiency is not equally probable in the judgment of tints by the eye. Conversely, if two persons, who are equally good judges, describe their impressions of a certain tint, and one says that it contains 4 portions of white and the other that it contains 16 portions, the most reasonable conclusion is that it really contains 8 portions. The arithmetic mean of the two estimates is 10, which is *not* the most probable value; it is the geometric mean 8, $(4:8::8:16)$, which is the most probable.

Precisely the same condition characterises every determination by each of the senses; for example, in judging of the weight of bodies or of their temperatures, of the loudness and of the pitches of tones, and of estimates of lengths and distances *as wholes*. Thus, three rods of the lengths a, b, c, when taken successively in the hand, appear to differ by equal intervals when $a:b::b:c$, and not when $a-b=b-c$. In all physiological phenomena, where there is on the one hand a stimulus and on the other a response to that stimulus Weber's or some other geometric law may be assumed to prevail in other words, the true mean is geometric rather than arithmetic.

The geometric mean appears to be equally applicable to the majority of the influences, which, combined with those of purely vital phenomena, give rise to the events with which sociology deals. It is difficult to find terms sufficiently general to apply to the varied topics of sociology, but there are two categories which are of common occurrence in which the geometric mean is certainly appropriate. The one is increase, as exemplified by the growth of population, where an already large nation tends to receive larger accessions than a small one under similar circumstances, or when a capital employed in a business increases in proportion to its size. The other category is the influences of circumstances or of "milieux" as they are often called, such as a period of plenty in which a larger field or a larger business yields a greater excess over its mean yield than a smaller one. Most of the causes of those differences with which sociology are concerned, and which are not purely vital phenomena, such as those previously discussed, may be classified under one or other of these two categories, or under such as are in principle almost the same. In short, sociological phenomena, like vital phenomena are, as a general rule, subject to the condition of the geometric mean.

The ordinary law of Frequency of Error, based on the arithmetic mean, corresponds, no doubt, sufficiently well with the observed facts of vital and social phenomena, to be very serviceable to statisticians, but it is far from satisfying their wants, and it may lead to absurdity when applied to wide deviations. It asserts that deviations in excess must be balanced by deviations of equal magnitude in deficiency; therefore, if the former be greater than the mean itself, the latter must be less than zero, that is, must be negative. This is an impossibility in many cases, to which the law is nevertheless applied by statisticians with no small success, so long as they are content to confine its application within a narrow range of deviation. Thus, in respect of Stature, the law is very correct in respect to ordinary measurements, although it asserts that the existence of giants, whose height is more than double the mean height of their race, implies the possibility of the existence of dwarfs, whose stature is less than nothing at all.

It is therefore an object not only of theoretical interest but of practical use, to thoroughly investigate a Law of Error, based on the geometric mean, even though some of the expected results may perhaps be apparent at first sight. With this view I placed the foregoing remarks in Mr. Donald Macalister's hands, who contributed a memoir that will be found in the *Proc. Royal Soc.*, No. 198, 1879, following my own. It should be referred to by such mathematicians as may read this book.

F.

PROBABLE EXTINCTION OF FAMILIES.[1]

The decay of the families of men who occupied conspicuous positions in past times has been a subject of frequent remark, and has given rise to various conjectures. It is not only the families of men of genius or those of the aristocracy who tend to perish, but it is those of all with whom history deals, in any way, even such men as the burgesses of towns, concerning whom Mr. Doubleday has inquired and written. The instances are very numerous in which surnames that were once common have since become scarce or have wholly disappeared. The tendency is universal, and, in explanation of it, the conclusion has been hastily drawn that a rise in physical comfort and intellectual capacity is necessarily accompanied by diminution in "fertility"—using that phrase in its widest sense and reckoning abstinence from marriage as one cause of sterility. If that conclusion be true, our population is chiefly maintained through the "proletariat," and thus a large element of degradation is inseparably connected with those other elements which tend to ameliorate the race. On the other hand, M. Alphonse de Candolle has directed attention to the fact that, by the ordinary law of chances, a large proportion of families are continually dying out, and it evidently follows that, until we know what that proportion is, we cannot estimate whether any observed diminution of surnames among the families whose history we can trace, is or is not a sign of their diminished "fertility." I give extracts from M. De Candolle's work in a foot-note,[2] and may add that, although I have not hitherto published anything on the matter, I took considerable pains some years ago to obtain numerical results in respect to this

[1] Reprinted, with slight revision, from the *Journ. Anthropol. Inst.* 1888.

[2] "Au milieu des renseignements précis et des opinions très-sensées de MM. Benoiston de Châteauneuf, Galton, et autres statisticiens, je n'ai pas rencontré la réflexion bien importante qu'ils auraient dû faire de l'extinction *inévitable* des noms de famille. Évidemment tous les noms doivent s'éteindre . . . . Un mathématicien pourrait calculer comment la réduction des noms ou titres aurait lieu, d'après la probabilité des naissances toutes féminines ou toutes masculines ou mélangées et la probabilité d'absence de naissances dans un couple quelconque," &c.—Alphonse de Candolle, *Histoire des Sciences et des Savants*, 1873.

very problem. I made certain very simple and not very inaccurate suppositions concerning average fertility, and I worked to the nearest integer, starting with 10,000 persons, but the computation became intolerably tedious after a few steps, and I had to abandon it. The Rev. H. W. Watson kindly, at my request, took the problem in hand, and his results form the subject of the following paper. They do not give what can properly be called a general solution, but they do give certain general results. They show (1) how to compute, though with great labour, any special case; (2) a remarkably easy way of computing those special cases in which the law of fertility approximates to a certain specified form; and (3) how all surnames tend to disappear.

The form in which I originally stated the problem is as follows. I purposely limited it in the hope that its solution might be more practicable if unnecessary generalities were excluded:—

A large nation, of whom we will only concern ourselves with the adult males, N in number, and who each bear separate surnames, colonise a district. Their law of population is such that, in each generation, a_0 per cent. of the adult males have no male children who reach adult life; a_1 have one such male child; a_2 have two; and so on up to a_5, who have five. Find (1) what proportion of the surnames will have become extinct after r generations; and (2) how many instances there will be of the same surname being held by m persons.

Discussion of the problem by the Rev. H. W. Watson, D.Sc., F.R.S., *formerly Fellow of Trinity College, Cambridge.*

Suppose that at any instant all the adult males of a large nation have different surnames, it is required to find how many of these surnames will have disappeared in a given number of generations upon any hypothesis, to be determined by statistical investigations, of the law of male population.

Let, therefore, a_0 be the percentage of males in any generation who have no sons reaching adult life, let a_1 be the percentage that have one such son, a_2 the percentage that have two, and so on up to a_q, the percentage that have q such sons, q being so large that it is not worth while to consider the chance of any man having more than q adult sons—our first hypothesis will be that the numbers

a_0, a_1, a_2, etc., remain the same in each succeeding generation. We shall also, in what follows, neglect the overlapping of generations—that is to say, we shall treat the problem as if all the sons born to any man in any generation came into being at one birth, and as if every man's sons were born and died at the same time. Of course it cannot be asserted that these assumptions are correct. Very probably accurate statistics would discover variations in the values of a_0, a_1, etc., as the nation progressed or retrograded; but it is not at all likely that this variation is so rapid as seriously to vitiate any general conclusions arrived at on the assumption of the values remaining the same through many successive generations. It is obvious also that the generations must overlap, and the neglect to take account of this fact is equivalent to saying, that at any given time we leave out of consideration those male descendants, of any original ancestor who are more than a certain average number of generations removed from him, and compensate for this by giving credit for such male descendants, not yet come into being, as are not more than that same average number of generations removed from the original ancestors.

Let then $\frac{a_0}{100}$, $\frac{a_1}{100}$, $\frac{a_2}{100}$, etc., up to $\frac{a_q}{100}$ be denoted by the symbols t_0, t_1, t_2, etc., up to t_q, in other words, let t_0, t_1, etc., be the chances in the first and each succeeding generation of any individual man, in any generation, having no son, one son, two sons, and so on, who reach adult life. Let N be the original number of distinct surnames, and let ${}_rm_s$ be the fraction of N which indicates the number of such surnames with s representatives in the rth generation.

Now, if any surname have p representatives in any generation, it follows from the ordinary theory of chances that the chance of that same surname having s representatives in the next succeeding generation is the coefficient of x^s in the expansion of the multinomial

$$(t_0 + t_1x + t_2x^2 + \text{, etc. } + t_qx^q)^p$$

Let then the expression $t_0 + t_1x + t_2x^2 +$ etc. $+ t_qx^q$ be represented by the symbol T.

Then since, by the assumption already made, the number of surnames with no representative in the r–1th generation is ${}_{r-1}m_0$ N, the number with one representative ${}_{r\text{-}1}m_1.\mathrm{N}$, the number with two ${}_{r\text{-}1}m_2.$ N and so on, it follows, from what we last stated, that the number of surnames with s representatives in the rth generation must be the coefficient of x^s in the expression

$$\left\{ {}_{r\text{-}1}m_0 + {}_{r\text{-}1}m_1\mathrm{T} + {}_{r\text{-}1}m_2\mathrm{T}^2 + \text{ etc. } + {}_{r\text{-}1}m_{q^{r\text{-}1}}\mathrm{T}^{q^{r-1}} \right\} \mathrm{N}$$

If, therefore, the coefficient of N in this expression be denoted by $f_r\ (x)$ it follows that ${}_{r\text{-}1}m_1$, ${}_{r\text{-}1}m_2$ and so on, are the coefficients of x, x^2 and so on, in the expression $f_{r\text{-}1}\ (x)$.

If, therefore, a series of functions be found such that

$$f_1(x) = t_0 + t_1 x + \text{ etc. } + t_q x^q \text{ and } f_r\ (x) = f_{r\text{-}1}\ (t_0 + t_1 x \text{ etc. } + t^q x^q)$$

then the proportional number of groups of surnames with s representatives in the rth generation will be the coefficient of x^s in $f_r\ (x)$ and the actual number of such surnames will be found by multiplying this coefficient by N. The number of surnames unrepresented or become extinct in the rth generation will be found by multiplying the term independent of x in $f_r\ (x)$ by the number N.

The determination, therefore, of the rapidity of extinction of surnames, when the statistical data, t_0, t_1, etc., are given, is reduced to the mechanical, but generally laborious process of successive substitution of $t_0 + t_1 x + t_2 x^2 +$ etc., for x in successively determined values of $f_r\ (x)$, and no further progress can be made with the problem until these statistical data are fixed; the following illustrations of the application of our formula are, however, not without interest.

(1) The very simplest case by which the formula can be illustrated is when $q = 2$ and t_0, t_1, t_2 are each equal to $\frac{1}{3}$.

$$\text{Here } f_1\ (x) = \frac{1 + x + x^2}{3} f_2(x) = \frac{1}{3}\left\{ 1 + \frac{1}{3}\ (1 + x + x^2) + \frac{1}{9}\ 1 + x + x^2)^2 \right\}^2$$

and so on.

Making the successive substitutions, we obtain

$$f_2\ (x) = \frac{1}{3}\ \left\{ \frac{13}{9} + \frac{5x}{9} + \frac{6x^3}{9} + \frac{2x}{9} + \frac{x}{9} \right\}$$

$$f_3(x) = \frac{1249}{2187} + \frac{265x}{2187} + \frac{343x^2}{2187} + \frac{166x^3}{2187} + \frac{109x^4}{2187} + \frac{34x^5}{2187} + \frac{16x^6}{2187} + \frac{4x^7}{2187} + \frac{x^8}{2187}$$

$$\begin{aligned} f_4(x) = {}& \cdot 63183 + \cdot 08306x + \cdot 10635x^2 + \cdot 07804x^3 + \cdot 06489x^4 + \cdot 05443x^5 + \cdot 01437x^6 \\ & + \cdot 01692x^7 + \cdot 01144x^8 + \cdot 00367x^9 + \cdot 00104x^{10} + \cdot 00015x^{11} + \cdot 00005x^{12} \\ & + \cdot 00001x^{13} + \cdot 00000x^{14} + \cdot 00000x^{15} + \cdot 00000x^{16} \end{aligned}$$

and the constant term in $f_5(x)$ or ${}_5m_0$ is therefore

$$6{\cdot}3183+\frac{{\cdot}08306}{3}+\frac{{\cdot}10635}{9}+\frac{{\cdot}07804}{27}+\frac{{\cdot}06489}{81}+\frac{{\cdot}05443}{243}+\frac{{\cdot}01437}{729}+\frac{{\cdot}01692}{2187}+\frac{{\cdot}01144}{6561}$$
$$+\frac{{\cdot}00367}{19683}+\frac{{\cdot}00104}{59049}+\frac{{\cdot}00015}{177147}+$$

The value of which to five places of decimals is ·67528.

The constant terms, therefore, in f_1, f_2 up to f_5 when reduced to decimals, are in this case ·33333, ·48148, ·57110, ·64113, and ·65628 respectively. That is to say, out of a million surnames at starting, there have disappeared in the course of one, two, etc., up to five generations, 333333, 481480, 571100, 641130, and 675280 respectively.

The disappearances are much more rapid in the earlier than in the later generations. Three hundred thousand disappear in the first generation, one hundred and fifty thousand more in the second, and so on, while in passing from the fourth to the fifth, not more than thirty thousand surnames disappear.

All this time the male population remains constant. For it is evident that the male population of any generation is to be found by multiplying that of the preceding generation, by t_1+2t_2, and this quantity is in the present case equal to one.

If axes Ox and Oy be drawn, and equal distances along Ox represent generations from starting, while two distances are marked along every ordinate, the one representing the total male population in any generation, and the other the number of remaining surnames in that generation, of the two curves passing through the extremities of these ordinates, the *population* curve will, in this case, be a straight line parallel to Ox, while the *surname* curve will intersect the population curve on the axis of y, will proceed always convex to the axis of x, and will have the positive part of that axis for an asymptote.

The case just discussed illustrates the use to be made of the general formula, as well as the labour of successive substitutions, when the expressions $f_1(x)$ does not follow some assigned law. The calculation may be infinitely simplified when such a law can be found; especially if that law be the expansion of a binomial, and only the extinctions are required.

For example, suppose that the terms of the expression t_0+t_1x+ etc. $+t_qx_q$ are proportional to the terms of the expanded binomial

$(a+bx)^q$ *i.e.* suppose that $t_0 = \frac{a^q}{(a+b)^q}$, $t_1 = q\frac{a^{q-1}b}{(a+b)^q}$ and so on.

$$\text{Here } f_1(x) = \frac{(a+bx)^q}{(a+b)^q} \text{ and } {}_1m_0 = \frac{a^q}{(a+b)^q}$$

$$f_2(x) = \frac{1}{(a+b)^q}\left\{ a + b\frac{(a+bx)^q}{(a+b)^q} \right\}^q$$

$${}_2m_0 = \frac{1}{(a+b)^q}\left\{ a + b\,{}_1m_0 \right\}^q$$

$$\text{Generally } {}_rm_0 = \frac{1}{(a+b)^q}\left\{ a + b_{r-1}m_0 \right\}^q = \frac{b_q}{(a+b)_q}\left\{ \frac{a}{b} + {}_{r-1}m_0 \right\}^q$$

If, therefore, we wish to find the number of extinctions in any generation, we have only to take the number in the preceding generation, add it to the constant fraction $\frac{a}{b}$, raise the sum to the power of q, and multiply by $\frac{b^q}{(a+b)^q}$

With the aid of a table of logarithms, all this may be effected for a great number of generations in a very few minutes. It is by no means unlikely that when the true statistical data t_0, t_1, etc., t_q are ascertained, values of a, b, and q may be found, which shall render the terms of the expansion $(a+bx)^q$ approximately proportionate to the terms of $f_1(x)$. If this can be done, we may *approximate* to the determination of the rapidity of extinction with very great ease, for any number of generations, however great.

For example, it does not seem very unlikely that the value of q might be 5, while $t_0, t_1 \dots t_q$ might be ·237, ·396, ·264, ·088, ·014, ·001, or nearly, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{7}{24}$, $\frac{1}{23}$, $\frac{1}{138}$, and $\frac{1}{1000}$.

Should that be the case, we have, $f_1(x) = \frac{(3+x)^5}{} \quad {}_1m_0 = \frac{3^5}{4^5}$

and generally $${}_rm_0 = \frac{1}{4^5}\left\{ 3 + {}_{r-1}m_0 \right\}^5$$

Thus we easily get for the number of extinctions in the first ten generations respectively.

·237, ·346, ·410, ·450, ·477, ·496, ·510, ·520, ·527, ·533.

We observe the same law noticed above in the case of $\frac{1+x+x^2}{3}$ viz., that while 237 names out of a thousand disappear in the first step, and an additional 109 names in the second step, there are only 27 disappearances in the fifth step, and only six disappearances in the tenth step.

If the curves of surnames and of population were drawn from this case, the former would resemble the corresponding curve in the case last mentioned, while the latter would be a curve whose distance from the axis of x increased indefinitely, inasmuch as the expression

$$t_1 + 2t_2 + 3t_3 + 4t_4 + 5t_5$$

is greater than one.

Whenever $f_1\ (x)$ can be represented by a binomial, as above suggested, we get the equation

$$_rm_0 = \frac{1}{(a+b)^q} \left\{ a + b_{r\text{-}1}m \right\}^q$$

whence it follows that as r increases indefinitely the value of $_rm_0$ approaches indefinitely to the value y where

$$y = \frac{1}{(a+b)} \left\{ a + by \right\}$$

that is where $y = 1$.

All the surnames, therefore, tend to extinction in an indefinite time, and this result might have been anticipated generally, for a surname once lost can never be recovered, and there is an additional chance of loss in every successive generation. This result must not be confounded with that of the extinction of the male population; for in every binomial case where q is greater than 2, we have $t_1 + 2t_2 +$ etc. $+ qt_q > 1$, and, therefore an indefinite increase of male population.

The true interpretation is that each of the quantities, $_rm_1$, $_rm_2$, etc., tends to become zero, as r is indefinitely increased, but that it does not follow that the product of each by the infinitely large number N is also zero.

As, therefore, time proceeds indefinitely, the number of surnames extinguished becomes a number of the *same order of magnitude* as the total number at first starting in N, while the number of surnames represented by one, two, three, etc., representatives is some infinitely smaller but finite number. When the finite numbers are multiplied by the corresponding number of representatives, sometimes infinite in number, and the products added together, the sum will generally exceed the original number N. In point of fact, just as in the cases calculated above to five generations, we had a continual, and indeed at first, a rapid extinction of surnames, combined in the one case with a stationary, and in the other case an increasing population, so is it when the number of generations is increased indefinitely. We have a continual extinction of surnames going on, combined with constancy, or increase of population, as the case may be, until at length the number of surnames remaining is absolutely insensible, as compared with the number at starting; but the total number of representatives of those remaining surnames is infinitely greater than the original number.

We are not in a position to assert from *actual calculation* that a corresponding result is true for every form of $f_1(x)$, but the reasonable inference is that such is the case, seeing that it holds whenever $f_1(x)$ may be compared with $\frac{(a+bx)^q}{(a+b)^q}$ whatever a, b, or q may be.

G.

ORDERLY ARRANGEMENT OF HEREDITARY DATA.

THERE are many methods both of drawing pedigrees and of describing kinship, but for my own purposes I still prefer those that I designed myself. The chief requirements that have to be fulfilled are compactness, an orderly and natural arrangement, and clearly intelligible symbols.

Nomenclature.—A symbol ought to be suggestive, consequently the initial letter of a word is commonly used for the purpose. But this practice would lead to singular complications in symbolizing the various ranks of kinship, and it must be applied sparingly. The letter F is equally likely to suggest any one of the three very different words of Father, Female, and Fraternal. The letter M suggests both Mother and Male; S would do equally for Son and for Sister. Whether they are English, French, or German words, much the same complexity prevails. The system employed in *Hereditary Genius* had the merit of brevity, but was apt to cause mistake; it was awkward in manuscript and difficult to the printer, and I have now abandoned it in favour of the method employed in the *Records* *of Family Faculties*. This will now be briefly described again. Each kinsman can be described in two ways, either by letters or by a number. In ordinary cases both the letter and number are intended to be used simultaneously, thus FF.8. means the Father's Father of the person described, though either FF or 8, standing by themselves, would have the same meaning. The double nomenclature has great practical advantages. It is a check against mistake and makes reference and orderly arrangement easy.

As regards the letters, F stands for Father and M for Mother, whenever no letter succeeds them; otherwise they stand for Father's and for Mother's respectively. Thus F is Father; FM is Father's Mother; FMF is Father's Mother's Father.

As regards the principle upon which the numbers are assigned, arithmeticians will understand me when I say that it is in accordance with the binary system of notation, which runs parallel to the binary distribution of the successive ranks of ancestry, as two parents, four grandparents, eight great-grandparents, and so on. The "subject" of the pedigree is of generation O; that of his parents, of generation 1; that of his grandparents, of generation 2, &c. This is clearly shown in the following table:—

Kinship.	Order of Generation.	Numerical Values							
		in Binary Notation.				in Decimal Notation.			
Child.............	0	1				1			
Parents	1	10		11		2		3	
Gr. Par............	2	100	101	110	111	4	5	6	7
G. Gr. Par.	3	1000	1010	1100	1110	8	10	12	14
		1001	1011	1100	1111	9	11	13	15

All the male ancestry are thus described by even numbers and the female ancestry by odd ones. They run as follows:—

F, 2.		M, 3.	
FF, 4.	FM, 5.	MF, 6.	MM, 7.
FFF, 8.	FMF, 10.	MFF, 12.	MMF, 14.
FFM, 9.	FMM, 11.	MFM, 13.	MMM, 15.

It will be observed that the double of the number of any ancestor is that of his or her Father; and that the double of the number *plus* 1 is that of his or her Mother; thus FM 5 has for her father FMF 10, and for her mother FMM 11.

When the word Brother or Sister has to be abbreviated it is safer not to be too stingy in assigning letters, but to write *br*, *sr*, and in the plural *brs*, *srs*; also for the long phrase of "brothers and sisters," to write *brss*.

All these symbols are brief enough to save a great deal of space, and they are perfectly explicit. When such a phrase has to be expressed as "the Fraternity of whom FF is one" I write in my own notes simply FF′, but there has been no occasion to adopt this symbol in the present book.

I have not satisfied myself as to any system for expressing descendants. Theoretically, the above binary system admits of extension by the use of negative indices, but the practical application of the idea seems cumbrous.

We and the French sadly want a word that the Germans possess to stand for Brothers and Sisters. Fraternity refers properly to the brothers only, but its use has been legitimately extended here to mean the brothers and the sisters, after the qualities of the latter have been reduced to their male equivalents. The Greek word *adelphic* would do for an adjective.

Pedigrees.—The method employed in the *Record of Family Faculties* for entering all the facts concerning each kinsman in a methodical manner was fully described in that book, and could not easily be epitomised here; but a description of the method in which the manuscript extracts from the records have been made for my own use will be of service to others when epitomising their own family characteristics. It will be sufficient to describe the quarto books that contain the medical extracts. Each page is ten and a half inches high and eight and a half wide, and the two pages, 252, 253, that are

SCHEDULE.

<table>
<tr><td colspan="6">JAMES LIPHOOK.</td><td colspan="6">JAMES LIPHOOK.</td></tr>
<tr><td colspan="3">Father's Father's Father and his fraternity.</td><td colspan="3">Father's Mother's Father and his fraternity.</td><td colspan="3">Mother's Father's Father and his fraternity.</td><td colspan="3">Mother's Mother's Father and his fraternity.</td></tr>
<tr><td colspan="3">Father's Father's Mother and her fraternity.</td><td colspan="3">Father's Mother's Mother and her fraternity.</td><td colspan="3">Mother's Father's Mother and her fraternity.</td><td colspan="3">Mother's Mother's Mother and her fraternity.</td></tr>
<tr><td colspan="3">Father's Father and his fraternity.</td><td colspan="3">Father's Mother and her fraternity.</td><td colspan="3">Mother's Father and his fraternity.</td><td colspan="3">Mother's Mother and her fraternity.</td></tr>
<tr><td colspan="2">Spare space.</td><td colspan="2">Father and his fraternity.</td><td colspan="2">Spare space.</td><td colspan="2">Spare space.</td><td colspan="2">Mother and her fraternity.</td><td colspan="2">Spare space.</td></tr>
<tr><td colspan="6">Spare space.</td><td colspan="6">Children.</td></tr>
</table>

EXAMPLE A.

Father's name . . . . JAMES GLADDING.
Mother's maiden name . . . . MARY CLAREMONT.

Initials.	Kin.	Principal illnesses and cause of death.	Age at death.
J. G.	Father	Bad rheum. fever; agonising headaches; diarrhœa; bronchitis; pleurisy. . *Heart disease*	54
R. G.	bro.	Rheum. gout . . . . *Apoplexy*	56
W. G.	bro.	Good health except gout; paralysis later *Apoplexy*	83
F. L.	sis.	Rheum. fever and rheum. gout . . . *Apoplexy*	73
C. G.	sis.	Delicate . . . . (inoculated) *Small pox*	
M. G.	Mother	Tendency to lung disease; biliousness; frequent heart attacks. *Heart disease and dropsy*	63
A. C.	bro.	Good health . . . . *Accident*	46
W. C.	bro.	Led a wild life . . . . . . *Premature old age*	62
E. C.	bro.	Always delicate . . . . *Consumption*	19
F. R.	sis.	Small-pox three times . . . . *General failure*	85
R. N.	sis.	Bilious; weak health . . . . *Cancer*	50
L. C.	sis.	. . . . *Fever*	21
M. G.	bro.	Inflam. lungs; rheum. fever . . *Heart disease*	17
K. G.	bro.	Debility; heart disease; colds . *Consumption*	40
G. L.	sis.	Bad headaches; coughs; weak spine; hysteria; apoplexy . . . . *Paralysis*	50
F. S.	sis.	Bad colds; inflam. lungs; hysteria	living
R. F.	sis.	Infantile paralysis; *colds*; nervous depression.	living
L. G.	sis.	Inflam. brain, also lungs; neuralgia; nervous fever	living

(Space left for remarks.)

EXAMPLE B.

Father's name . . . . JULIUS FITZROY.
Mother's maiden name . . . AMELIA MERRYWEATHER.

Initials.	Kin.	Principal illnesses and cause of death.	Age at death.
R. F.	Father	Gouty Habit . . *Weak Heart and Congest. Liver*	73
L. F.	bro.	. . . . *Gout and Decay*	88
A. G. F.	bro.	. . . . *Accident*	48
W. F.	bro.	. . . . *Typhoid*	16
	Mother	Gall stones . . . . *Internal Malady* (?) *Cancer*	55
P. M.	bro.	. . . . *Paralysis*	86
A. M.	bro.	. . . . *Paralysis*	85
N. M.	bro.	Still living.	
R. B.	sis.	. . . . *Consumption*	33
C. M.	sis.	. . . . *Rheum. in Head*	88
F. L.	sis.	. . . . *Softening of Brain*	76
		1 died an infant.	
G. F.	bro.	Gout: tendency to mesenteric disease; eruptive disorders . . . *Blood poisoning after a cut*	46
H. F.	bro.	Liver deranged; bad headaches; once supposed consumptive . . . . . *Gradual Paralysis*	45
S. T. F.	bro.	Eruptive disorder; mesentery disease; inflammation of liver . . *Inflammation of Lungs*	42
H. G.	sis.	Eruptive disorder; liver . . . *Inflam. of Lungs*	47
H. B. R.	sis.	Delicate; tend. to mesent. disease . *Consumption*	29
N. F.	sis.	Colds; liver disorder . . . . . . *Consumption*	30
E. L. F.	sis.	Mesenteric disease; grandular swellings . *Atrophy*	16
		2 died infants.	

(Space left for remarks.)

found wherever the book is opened, relate to the same family. The open book is ruled so as to resemble the accompanying schedule, which is drawn on a reduced scale on page 251. The printing within the compartments of the schedule does not appear in the MS. books, it is inserted here merely to show to whom each compartment refers. It will be seen that the paternal ancestry are described in the left page, the maternal in the right. The method of arrangement is quite orderly, but not altogether uniform. To avoid an unpleasing arrangement like a tree with branches, and which is very wasteful of space, each grandparent and his own two parents are arranged in a set of three compartments one above the other. There are, of course, four grandparents and therefore four such sets in the schedule. Reference to the examples A and B pages 252 and 253 will show how these compartments are filled up. The rest of the Schedule explains itself. The children of the pedigree are written below the compartment assigned to the mother and her brothers and sisters; the spare spaces are of much occasional service, to receive the overflow from some of the already filled compartments as well as for notes. It is astonishing how much can be got into such a schedule by writing on ruled paper with the lines one-sixth of an inch apart, which is not too close for use. Of course the writing must be small, but it may be bold, and the figures should be written very distinctly.

For a less ambitious attempt, including the grandparents and their fraternity, but not going further back, the left-hand page would suffice, placing "Children" where "Father" now stands, "Father's Father" for "Father," and so on throughout.